IT'S THE 2024 GENERAL ELECTION, BORIS JOHNSON VERSUS KEIR STARMER. JOHNSON SAYS HE IS 85% LIKELY TO CUT INCOME TAX IF ELECTED. STARMER SAYS HE IS 25% LIKELY TO CUT INCOME TAXES IF ELECTED. THE POLLS SAY THAT JOHNSON AND STARMER ARE EQUALLY LIKELY TO BE ELECTED. YOU FALL INTO A COMA ON ELECTION NIGHT AND WAKE UP TO FIND THAT INCOME TAX HAS BEEN CUT. WHAT IS THE LIKELIHOOD THAT JOHNSON WAS ELECTED?

$P(J) = 0.5$ $\quad P(S) = 0.5$

$P(\text{Cut} | \text{JOHNSON}) = 0.85$ $\quad P(\text{CUT} | \text{STARMER}) = 0.25$

$P(\text{Cut}' | \text{JOHNSON}) = 0.15$ $\quad P(\text{CUT}' | \text{STARMER}) = 0.75$

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$ IS BAYES THEOREM

WE KNOW THESE ALREADY

WE WANT $P(\text{Johnson} | \text{Cut})$

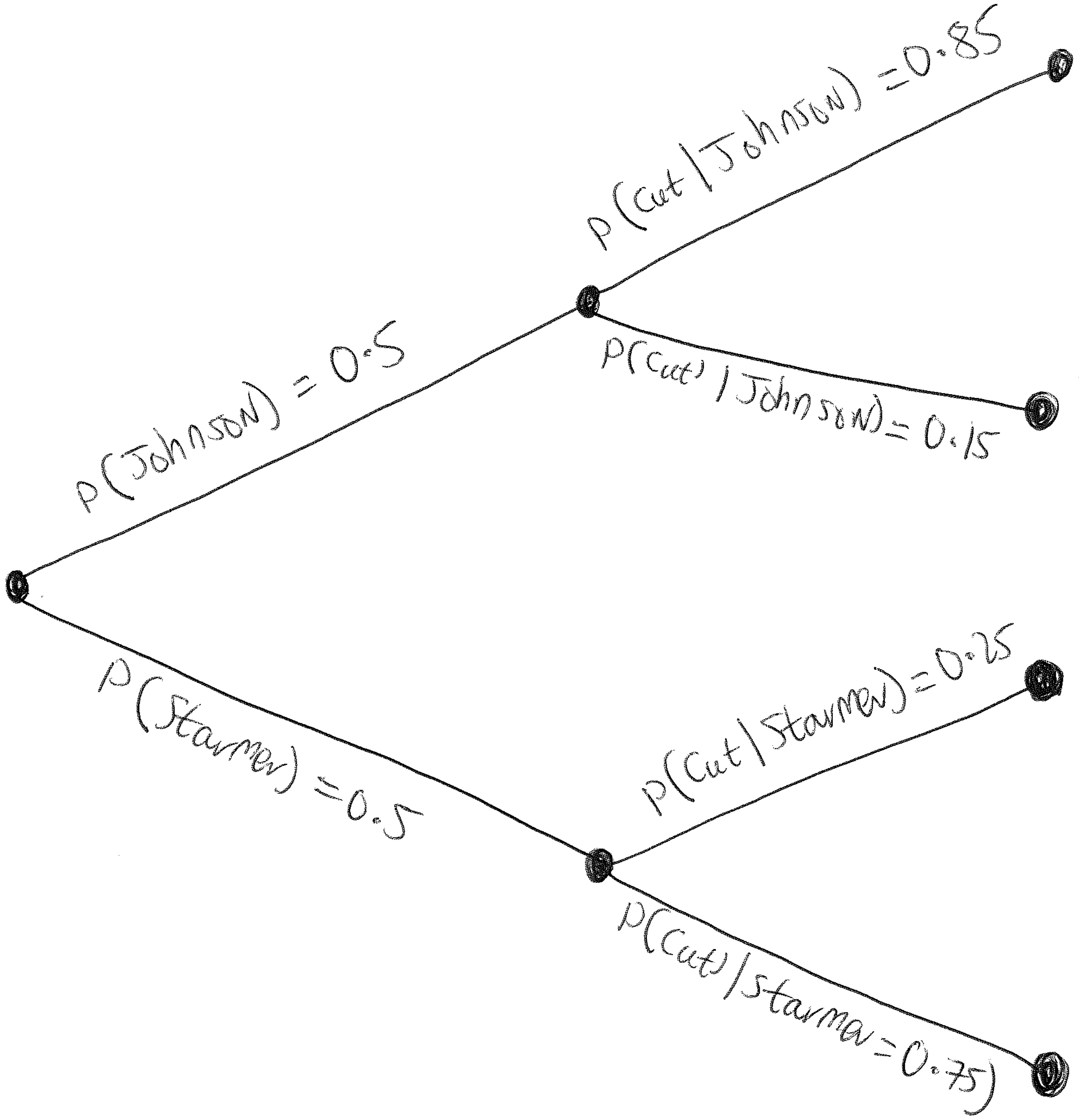
SO,
A = Johnson
B = Cut

$$\text{So } P(\text{Johnson} | \text{Cut}) = \frac{P(\text{Cut} | \text{Johnson}) \times P(\text{Johnson})}{P(\text{Cut})}$$

WE DON'T KNOW THIS YET, SO LET'S WORK IT OUT

...

WHAT IS P(Cut)? FOUR SCENARIOS

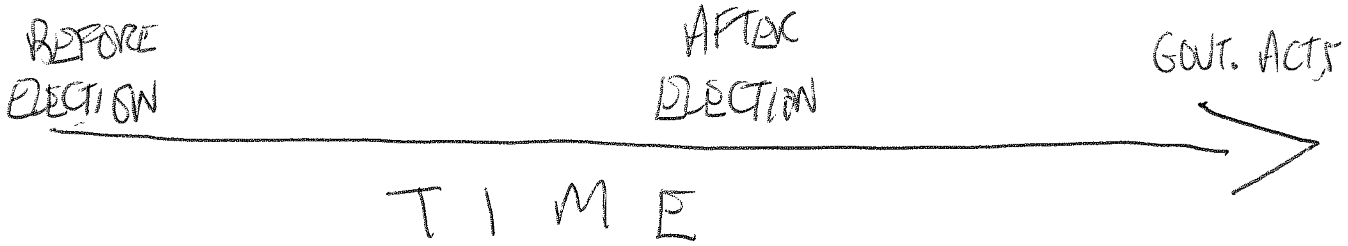


JOHNSON ELECTED AND HE CUTS TAX $= 0.5 \times 0.85 = \boxed{0.425}$

JOHNSON ELECTED AND DOESN'T CUT TAX $= 0.5 \times 0.15 = 0.075$

STARMER ELECTED AND HE CUTS TAX $= 0.5 \times 0.25 = \boxed{0.125}$

STARMER ELECTED AND HE DOESN'T CUT TAX $= 0.5 \times 0.75 = 0.375$



So $P(\text{Cut}) = 0.425 + 0.125 = 0.55$

$P(\text{Johnson}) = 0.5$ $\quad P(\text{Cut} \mid \text{Johnson}) = 0.85$ $\quad P(\text{Cut}) = 0.55$

BAYES SAYS $\quad P(A \mid B) = \dfrac{P(B \mid A) \times P(A)}{P(B)}$

So $P(\text{Johnson} \mid \text{Cut}) = \dfrac{P(\text{Cut} \mid \text{Johnson}) \times P(\text{Johnson})}{P(\text{Cut})}$

$$= \frac{0.85 \times 0.5}{0.55}$$

$$= 0.773 = 77\%$$

A DISEASE AFFECTS 0.1% OF POPULATION AND YOU TEST POSITIVE FOR IT. THE TEST GIVES A POSITIVE RESULT FOR 99% OF PATIENTS WHO HAVE THE DISEASE AND POSITIVE FOR 1% OF PATIENTS WHO DON'T HAVE THE DISEASE.

$p(dis) = 0.001$ $\quad p(pos|dis) = 0.99$

$p(dis') = 0.999$ $\quad p(pos|dis') = 0.01$

$$\boxed{P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}}$$

$$p(dis|pos) = \frac{p(pos|dis) \times P(dis)}{P(pos)}$$

WHAT IS $P(pos)$? ...ONLY TWO WAYS OF TESTING POSITIVE...

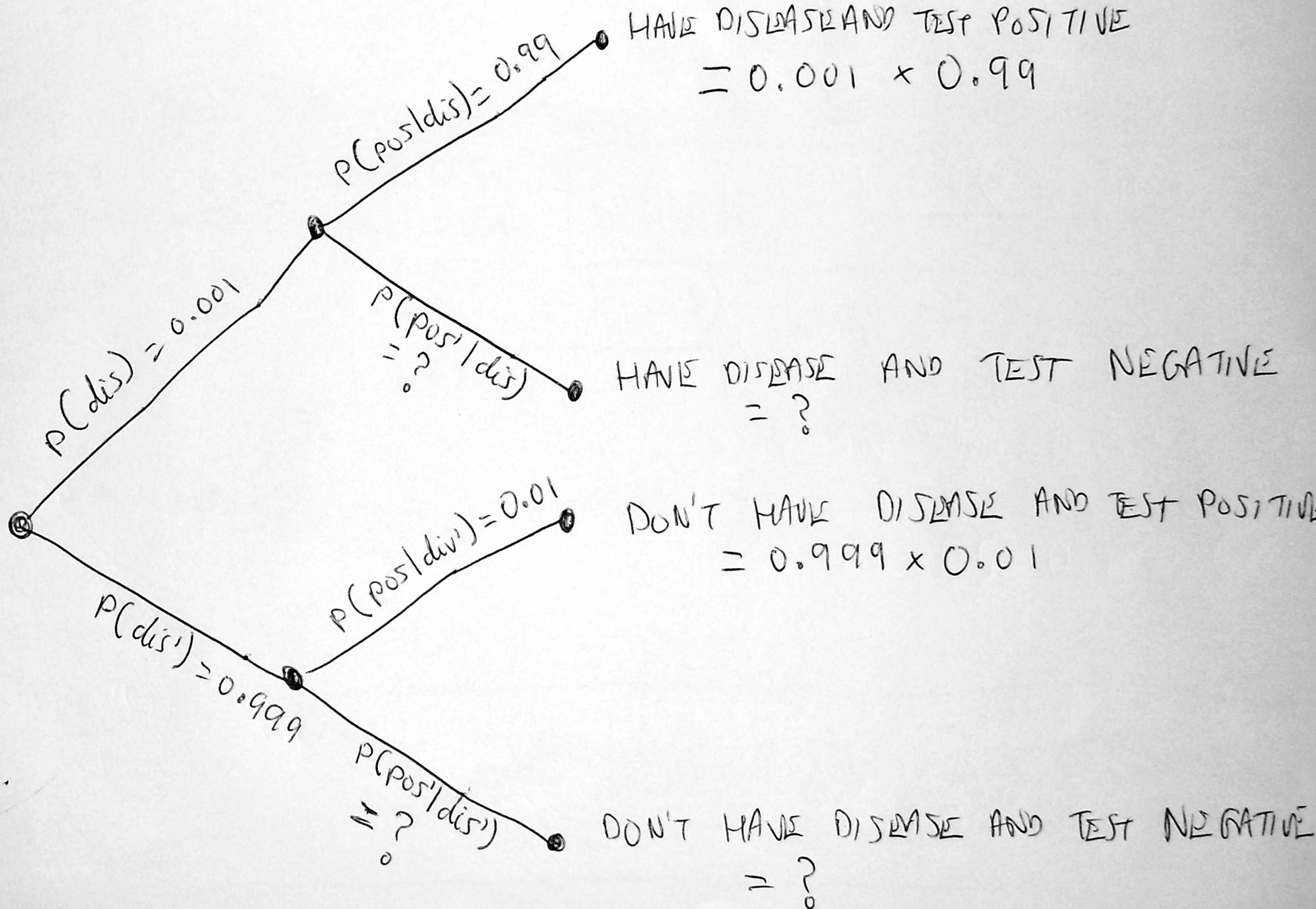
ONE: $p(dis) \times p(pos|dis)$ + TWO: $p(dis') \times P(pos|dis')$

$$p(dis|pos) = \frac{P(pos|dis) \times P(dis)}{p(dis) \times p(pos|dis) + p(dis') \times p(pos|dis')}$$

$$= \frac{0.99 \times 0.001}{0.001 \times 0.99 + 0.999 \times 0.01}$$

$$= 0.09$$

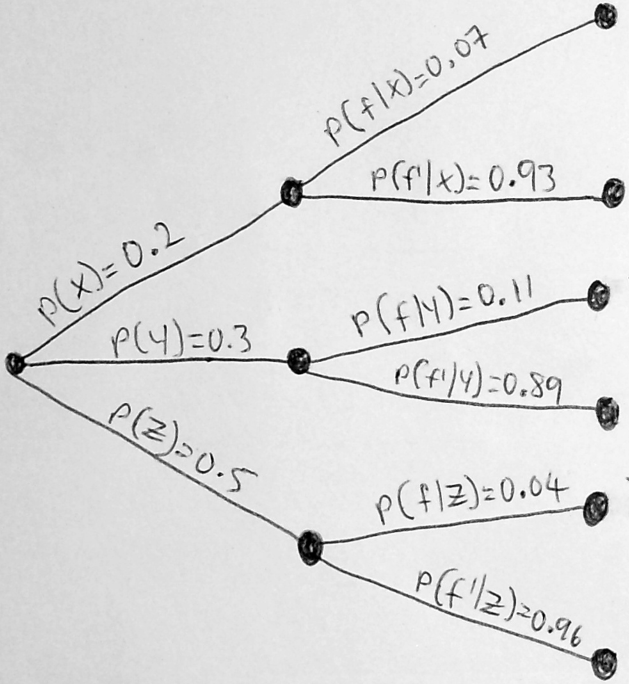
DISEASE



$$p(pos) = 0.001 \times 0.99 + 0.999 \times 0.01$$

DISEASE

THREE WRITERS ARE EACH PRODUCING SEVERAL 2,000-WORD BLOCKS A WEEK AND AT THE END OF A YEAR HAVE PRODUCED A COMBINED CORPUS OF WHICH X WROTE 20%, Y WROTE 30% AND Z WROTE 50%. THEY ALL USE LITERARY FEATURE f BUT AT DIFFERENT RATES. IN A 2,000-WORD BLOCK, X IS 7% LIKELY TO USE IT, Y IS 11% LIKELY TO USE IT, AND Z IS 4% LIKELY TO USE IT. A 2,000-WORD BLOCK CHOSEN AT RANDOM FROM THE CORPUS IS FOUND TO CONTAIN FEATURE f. WHAT IS THE LIKELIHOOD THAT WRITER Z WROTE BLOCK?



X WROTE THE BLOCK AND USED THE FEATURE = 0.014

X WROTE THE BLOCK AND DIDN'T USE FEATURE = 0.186

Y WROTE THE BLOCK AND USED THE FEATURE = 0.033

Y WROTE THE BLOCK AND DIDN'T USE FEATURE = 0.267

Z WROTE THE BLOCK AND USED THE FEATURE = 0.02

Z WROTE THE BLOCK AND DIDN'T USE FEATURE = 0.48

$$P(f) = 0.014 + 0.033 + 0.02 = 0.067$$

WRITING

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

$P(X) = 0.2$ $P(Y) = 0.3$ $P(Z) = 0.5$

$P(f|X) = 0.07$ $P(f|Y) = 0.11$ $P(f|Z) = 0.04$

$$P(f) = P(X) \times P(f|X) + P(Y) \times P(f|Y) + P(Z) \times P(f|Z)$$

$$P(f) = (0.2 \times 0.07) + (0.3 \times 0.11) + (0.5 \times 0.04)$$

$$P(f) = 0.067$$

FOR WRITER X

$$P(X|f) = \frac{P(f|X) \times P(X)}{P(f)}$$

$$P(X|f) = \frac{0.07 \times 0.2}{0.067}$$

$$= 20.9\%$$

FOR WRITER Y

$$P(Y|f) \quad \frac{P(f|Y) \times P(Y)}{P(f)}$$

$$= \frac{0.11 \times 0.3}{0.067}$$

$$= 49.3\%$$

FOR WRITER Z

$$P(Z|f) = \frac{P(f|Z) \times P(Z)}{P(f)}$$

$$= \frac{0.04 \times 0.5}{0.067}$$

$$= 29.9\%$$

WRITING